$$\therefore \frac{1+at+bu}{\sqrt{[1+a^2+b^2]}}=\frac{1+ct+du}{\sqrt{[1+c^2+d^2]}}=\frac{1+ft+gu}{\sqrt{[1+f^2+g^2]}} \text{ determines } t \text{ and } u.$$

$$e^2=\tan^2\theta=\frac{(t-a)^2+(u-b)^2+(au-bt)^2}{(1+at+bu)^2},\quad e_1^2=\frac{(t-a)^2+(u+b)^2+(au+bt)^2}{(1+at-bu)^2},$$

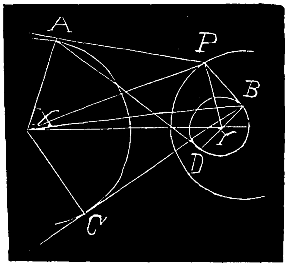
$$e_2^2=\frac{(t+a)^2+(u-b)^2+(au+bt)^2}{(1-at+bu)^2},\quad e_3^2=\frac{(t+a)^2+(u+b)^2+(au-bt)^2}{(1-at-bu)^2}.$$

145. Proposed by FRANK GIFFIN, Graduate Student, State University, Boulder, Col.

If A and B be the points of contact, upon two circles X and Y, of tangents drawn from any point of their circle of similitude, then the tangent from A to Y is equal to the tangent from B to X. [From *Casey's Sequel to Euclid*, Part I., page 144.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be any point on the circle of similitude, AP, BP the tangents from P to X and Y, respectively. AD, BC the tangents from A and B to Y and X, respectively.



Let $AX=R$, $BY=r$. Let $AD=a$, $BC=b$, $AP=c$, $BP=d$, $PX=m$, $PY=n$.

$\angle APX=\angle BPY$ since P is on circle of similitude. $\therefore$ $\angle APY=\angle BPX=\theta$.

Also, $c:d=R:r$. $\therefore$ $d=cr/R\ldots.(1)$.

$m:n=R:r$. $\therefore$ $n=mr/R\ldots.(2)$.

$$a^2=AY^2-r^2=c^2+n^2-2cn\cos\theta-r^2$$
$$=c^2+m^2r^2/R^2-(2cmr/R)\cos\theta-r^2\ldots.(3).$$

$$b^2=RX^2-R^2=d^2+m^2-2dm\cos\theta-R^2=c^2r^2/R^2+m^2-(2cmr/R)\cos\theta-R^2\,..(4).$$

(3)$-$(4) gives $R^2(a^2-b^2)=(c^2-m^2+R^2)(R^2-r^2)$.

But $c^2+R^2=m^2$.

$\therefore$ $a^2-b^2=0$. $\therefore$ $a=b$.

CALCULUS.

106. Proposed by M. C. STEVENS, M. A., Professor of Higher Mathematics, Purdue University, Lafayette, Ind.

$$\int_0^\pi \frac{\cos rx dx}{1-2a\cos x+a^2}=\frac{\pi a^r}{1-a^2}.$$

[*Williamson's Integral Calculus*, 6th Edition, page 174.]

Solution by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

In Todhunter's *Plane Trigonometry*, 3d Edition, Art. 309, we have

$$\cos\alpha+a\cos(\alpha+\beta)+a^2\cos(\alpha+2\beta)+\ldots\ a^{n-1}\cos[\alpha+(n-1)\beta]$$

$$=\frac{\cos\alpha-a\cos(\alpha-\beta)}{1-2a\cos\beta+a^2}\ldots.(1),$$

if n be indefinitely increased. Make $\alpha=0$. Then

$$1+a\cos\beta+a^2\cos2\beta+\ldots.a^{n-1}\cos[(n-1)\beta]=\frac{1-a\cos\beta}{1-2a\cos\beta+a^2}\ldots.(2).$$

Multiply (2) by 2 and subtract a unit from both members of the resulting equation, then

$$\frac{1-a^2}{1-2a\cos\beta+a^2}=1+2a\cos\beta+2a^2\cos2\beta+\ldots.2a^{n-1}\cos[(n-1)\beta]\ldots.(3).$$

$$\text{Then, }\int_0^\pi\frac{\cos rx dx}{1+2a\cos x+a^2}=\int_0^\pi\frac{\cos rx}{1-a^2}\{1+2a\cos x+2a^2\cos2x+\ldots.$$

$$+2a^{n-1}\cos[(n-1)x]\}dx=\frac{2a^r}{1-a^2}\int_0^\pi\cos^2 rx dx=\frac{\pi a^r}{1-a^2},$$

the terms of the series but this one vanishing between the given limits, as may be seen after reducing for a few terms.

Also solved by *G. B. M. ZERR*, *W. W. LANDIS*, *J. SCHEFFER*, and *L. C. PLANT*.

107. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College Mechanicsburg, Pa.**

The speed of signaling in submarine telegraph-cable varies as $x^2\log(1/x)$, in which x is the ratio of the radius of the core to that of the covering. Prove that the *maximum speed* is attained when this ratio is $1:\sqrt{e}$.

I. Solution by J. W. YOUNG, Cornell University, Ithaca, N. Y.; COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tenn.; and G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

$$\text{If } y=x^2\log\left(\frac{1}{x}\right)=-x^2\log x,\quad \frac{dy}{dx}=-2x\log x-x=-x(\log x^2+1).$$

$\log x^2+1=0$ gives the maximum.

$$\therefore\ \log\frac{1}{x^2}=1.\quad \therefore\ \frac{1}{x^2}=e,\text{ and } x=\frac{1}{\sqrt{e}}.$$

II. Solution by H. C. WHITAKER, Ph.D., Manual Training School, Philadelphia, Pa.; W. W. LANDIS, A.M., Dickinson College, Carlisle, Pa.; J. O. MAHONEY, B. E., M. Sc., Central High School, Dallas, Tex.; ALOIS F. KOVARIK, Decorah Institute, Decorah, Ia.; and J. JCHEFFER, A. M., Hagerstown, Md.

$$\text{If } S=cx^2\log\frac{1}{x},\quad \frac{dS}{dx}=2cx\log\frac{1}{x}-cx.$$